Apex Consumers and Trophic Downgrading

Andrea de Lima Ribeiro (Oc.)
Carlos Andrés Marcelo-Serván (Biol.)
Carolina Arruda Moreira (Phys.)
Cecilia Andreazzi (Biol.)
César Parra (Phys.)
Henrique Rubira (Undergrad.)

II Southern-Summer School on Mathematical Biology

Some concepts

- Apex (alpha, top-level) predators: no predators at their own;
- Apex consumers may be able to model an ecosystem [1];
- Concept of trophic cascades;
- Propagation of impacts by consumers on their prey downward through food webs. [3]

Some concepts

- Loss of Apex changes the species composition and density of lower levels;
- Top-down control is not the only process that modulate the species diversity.

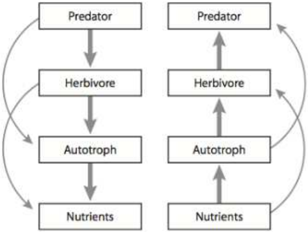
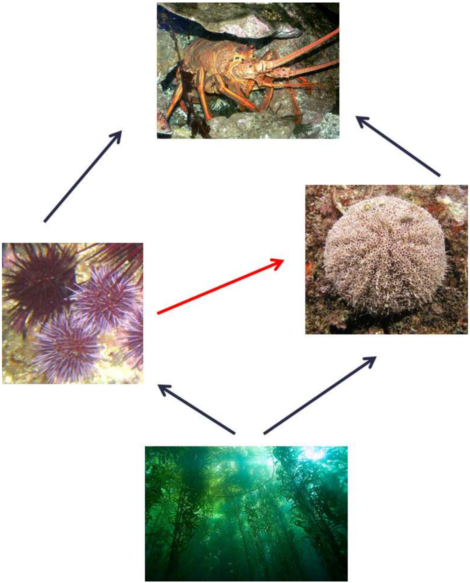


Figura: From [2]

The system



Populations




The article

Ecological Applications, 14(5), 2004, pp. 1566–1573

FISHING FOR LOBSTERS INDIRECTLY INCREASES EPIDEMICS IN SEA URCHINS

KEVIN D. LAFFERTY[1]

USGS Western Ecological Research Center, c/o Marine Science Institute, University of California, Santa Barbara, California 93106 USA

Objectives and questions

Objectives

To develop a mathematical model that describes the dynamics of the system that Lafferty (2004) studied.

Questions

- Is there evidence for top-down control? What are the conditions for its occurrance?
- How do trophic cascade and epidemics interact?
- How impacts on the apex predator population affects the food chain and epidemiological dynamics of this system?

Mathematical Model - Equations

$$\frac{dK}{dt} = K(1-K) - a(U_1+U_2)K$$
$$\frac{dU_1}{dt} = -b\frac{U_1}{U_1+U_2+U_c}L - m_0U_1 - gm_1U_1 + rU_1U_2$$
$$\frac{dU_2}{dt} = ac(U_1+U_2)K - b\frac{U_2}{U_1+U_2+U_c}L - m_0U_2 - rU_1U_2$$
$$\frac{dL}{dt} = bd\frac{U_1+U_2}{U_1+U_2+U_c}L - m_2L$$

K, U_1, U_2, L: populations of kelps, infected urchins, not infected urchins and lobsters

a, b: attack rates of urchins and lobsters

c, d: efficiency of predation by urchins and lobsters

r: transmission rate of infection

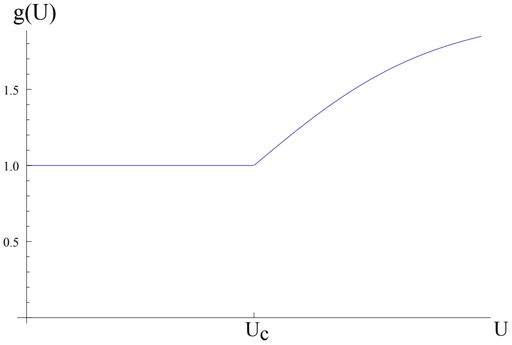
m_0, m_1, m_2: mortality rates of urchins (natural causes), urchins (infection) and lobsters

g: multiplying factor

U_c: urchin critical density

Mathematical Model - Equations

$$g(U) = 1 + \alpha \frac{1 - e^{-\beta(U - U_c)}}{1 + e^{-\beta(U - U_c)}}$$



$\alpha = 1.0$ $\beta = 5.0$

Results

Sub-system: kelps and not infected urchins

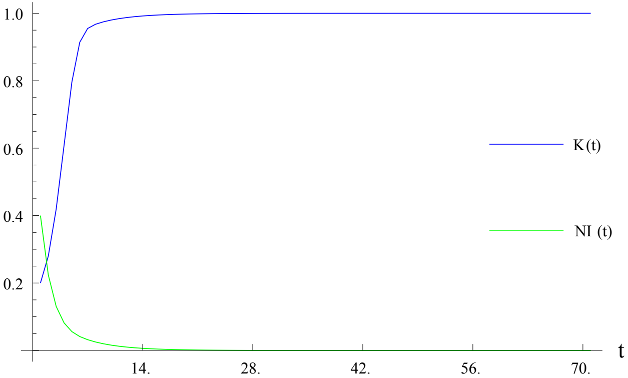
$$\frac{dK}{dt} = K(1-K) - aU_2K$$
$$\frac{dU_2}{dt} = acU_2K - m_0U_2$$

Important fixed points:

1. $K^* = 1$, ${U_2}^* = 0$: stable for $m_0 > ac$
2. $K^* = \frac{m_0}{ac}$, ${U_2}^* = \frac{ac - m_0}{a^2c}$: exists and it's stable for $m_0 < ac$

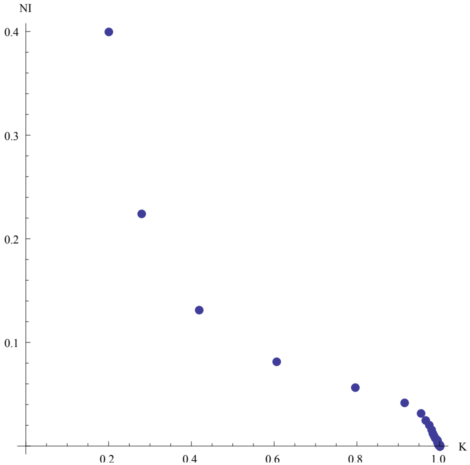
Results

$a = 1.0$, $c = 0.3$, $m_0 = 0.5$



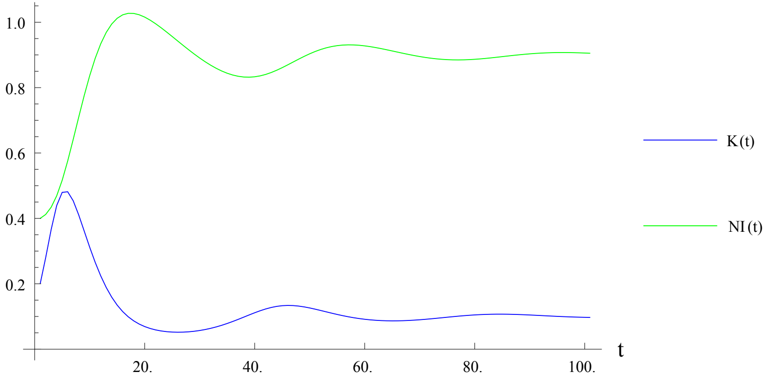
Results

$a = 1.0$, $c = 0.3$, $m_0 = 0.5$



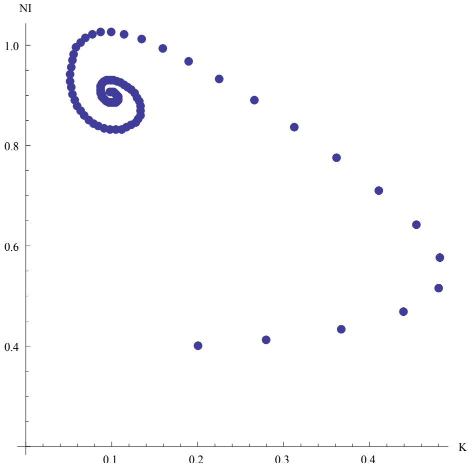
Results

$a = 1.0$, $c = 0.3$, $m_0 = 0.03$



Results

$a = 1.0$, $c = 0.3$, $m_0 = 0.03$



Results

Sub-system: kelps, infected and not infected urchins (lobsters have been removed)

$$\frac{dK}{dt} = K(1-K) - a(U_1+U_2)K$$
$$\frac{dU_1}{dt} = -m_0U_1 - gm_1U_1 + rU_1U_2$$
$$\frac{dU_2}{dt} = ac(U_1+U_2)K - m_0U_2 - rU_1U_2$$

Results

Important fixed points:

1. $K^* = \frac{m_0}{ac}$, ${U_1}^* = 0$, ${U_2}^* = \frac{ac - m_0}{a^2 c}$: stable for $r < \frac{a^2 c(m_0 + g m_1)}{ac - m_0}$
2.

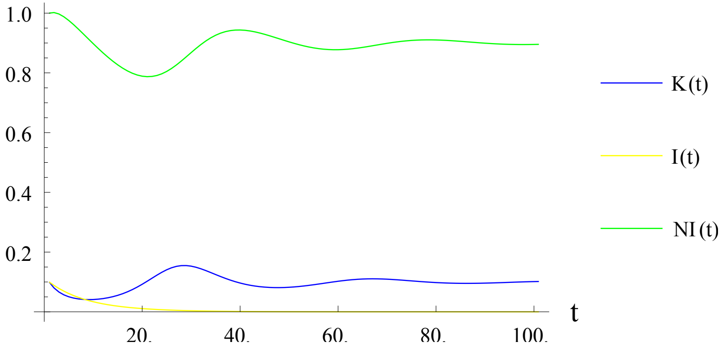
$$K^* = \frac{1}{2acr}\Big[r(ac + m_0 + g m_1) - \sqrt{r\left(r(ac + m_0 + g m_1)^2 + 4ac(m_0 + g m_1)(-r + a g m_1)\right)}\Big]$$

$${U_1}^* = \frac{1}{2a^2 cr}\Big[r(ac - m_0 - g m_1) - 2a^2 c(m_0 + g m_1) + \sqrt{r\left(r(ac + m_0 + g m_1)^2 + 4ac(m_0 + g m_1)(-r + a g m_1)\right)}\Big]$$

$${U_2}^* = \frac{m_0 + g m_1}{r}$$

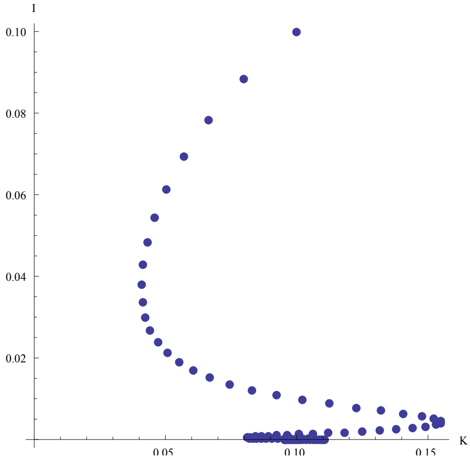
exists and it's stable for $r > \frac{a^2 c(m_0 + g m_1)}{ac - m_0}$

$r = 0.1$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$



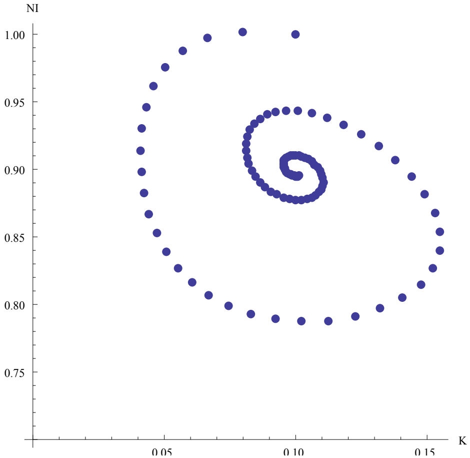
Results

$r = 0.1$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$



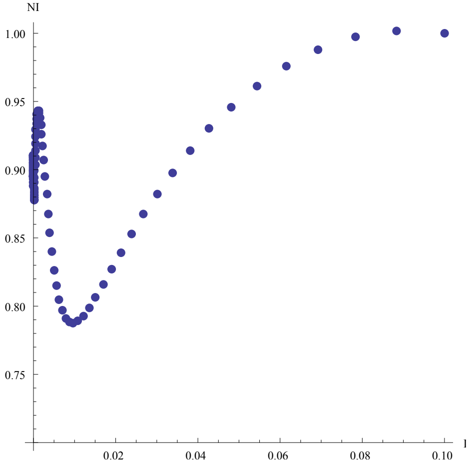
Results

$r = 0.1$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$



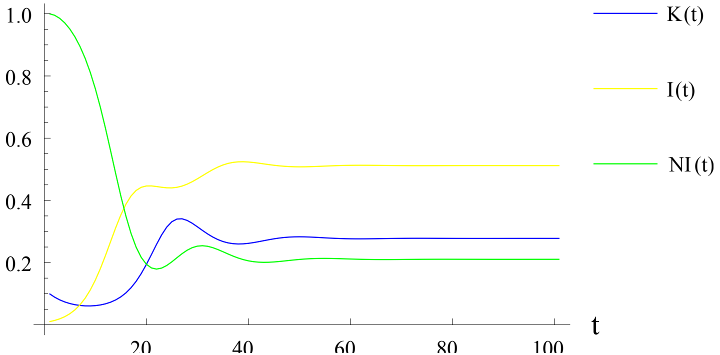
Results

$r = 0.1,\ \alpha = 1.0,\ m_1 = 0.05,\ U_c = 0.5$



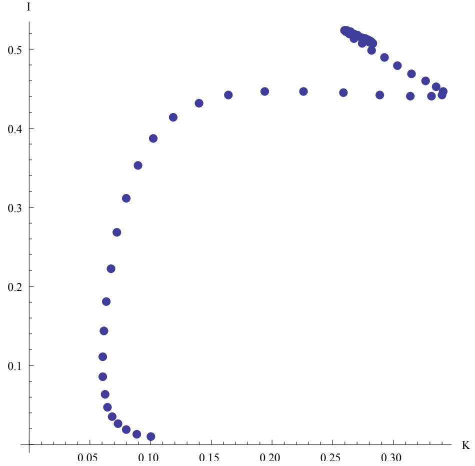
Results

$r = 0.5$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$



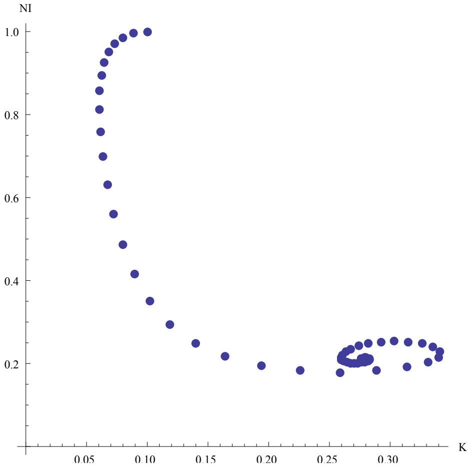
Results

$r = 0.5$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$



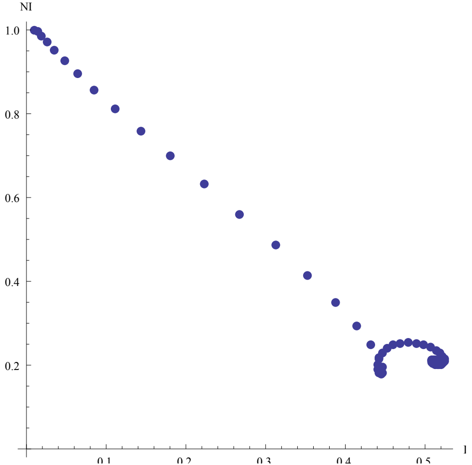
Results

$r = 0.5$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$

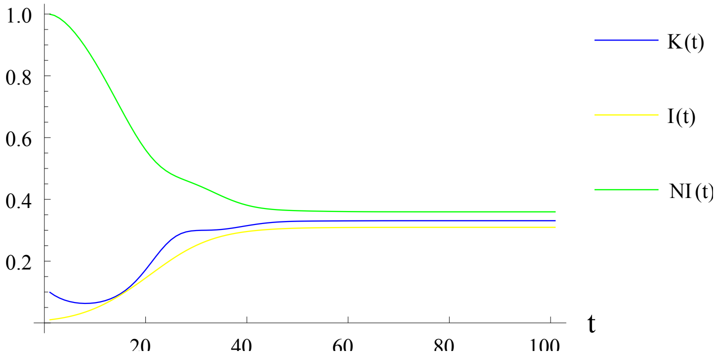


Results

$r = 0.5$, $\alpha = 1.0$, $m_1 = 0.05$, $U_c = 0.5$



Effect of increased mortality:
$r = 0.5$, $\alpha = 5.0$, $m_1 = 0.05$, $U_c = 0.5$



Results

All four populations (reintroduction of lobsters)

$$\frac{dK}{dt} = K(1-K) - a(U_1+U_2)K$$
$$\frac{dU_1}{dt} = -b\frac{U_1}{U_1+U_2+U_c}L - m_0U_1 - gm_1U_1 + rU_1U_2$$
$$\frac{dU_2}{dt} = ac(U_1+U_2)K - b\frac{U_2}{U_1+U_2+U_c}L - m_0U_2 - rU_1U_2$$
$$\frac{dL}{dt} = bd\frac{U_1+U_2}{U_1+U_2+U_c}L - m_2L$$

Results

Important fixed points:

1.

$$K^* = \frac{1}{2acr}\Big[r(ac + m_0 + gm_1) - \sqrt{-2acr^2(m_0+gm_1) + r^2(m_0+gm_1)^2 + a^2cr(cr + 4gm_1(m_0+gm_1))}\Big]$$

$$U_1{}^* = \frac{1}{2a^2cr}\Big[acr - 2a^2c(m_0+gm_1) - r(m_0+gm_1) + \sqrt{-2acr^2(m_0+gm_1) + r^2(m_0+gm_1)^2 + a^2\left(4crgm_0m_1 + cr\left(cr + 4g^2m_1{}^2\right)\right)}\Big]$$

$$U_2{}^* = \frac{m_0 + gm_1}{r}$$

$$L^* = 0$$

stable for $b < b_0$

$$b_0 = \frac{m_2}{d}\left(1 + \frac{a^2cU_c}{ac - m_0}\right)$$

Results

Important fixed points:

2

$$K^* = 1 - \frac{am_2 U_c}{bd - m_2}$$

$$U_1{}^* = 0$$

$$U_2{}^* = \frac{m_2 U_c}{bd - m_2}$$

$$L^* = \frac{1}{(bd - m_2)^2} dU_c \left((ac - m_0)(bd - m_2) - a^2 c m_2 U_c\right)$$

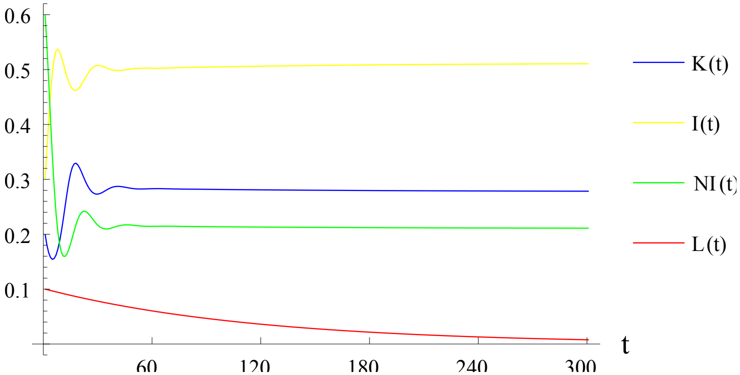
exists for $b > b_0$ and it's stable for $b_0 < b < b1$ with stable limit cycle for $b > b_1$

$$b_0 = \frac{m_2}{d}\left(1 + \frac{a^2 c U_c}{ac - m_0}\right)$$

b_1: no analytical expression

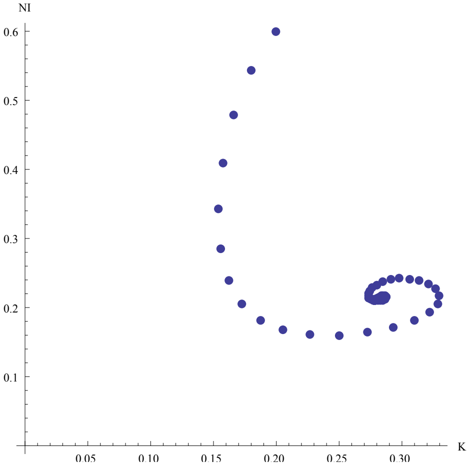
Results

$b = 0.05$, $d = 0.05$, $m_2 = 0.01$

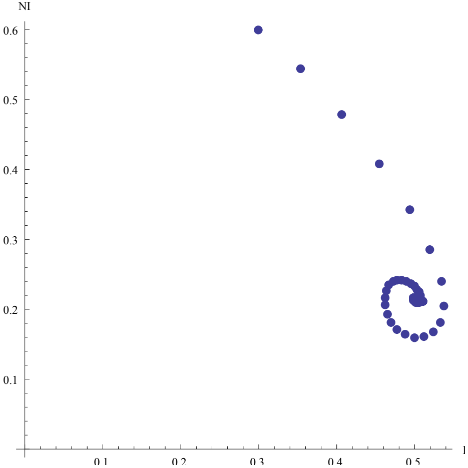


Results

$b = 0.05$, $d = 0.05$, $m_2 = 0.01$

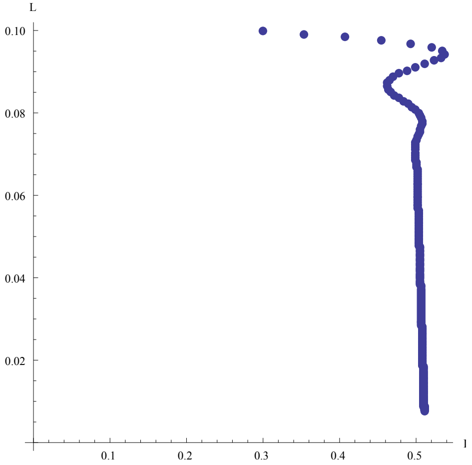


$b = 0.05$, $d = 0.05$, $m_2 = 0.01$

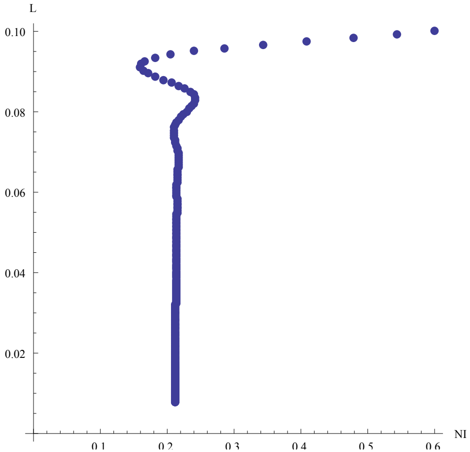


Results

$b = 0.05$, $d = 0.05$, $m_2 = 0.01$

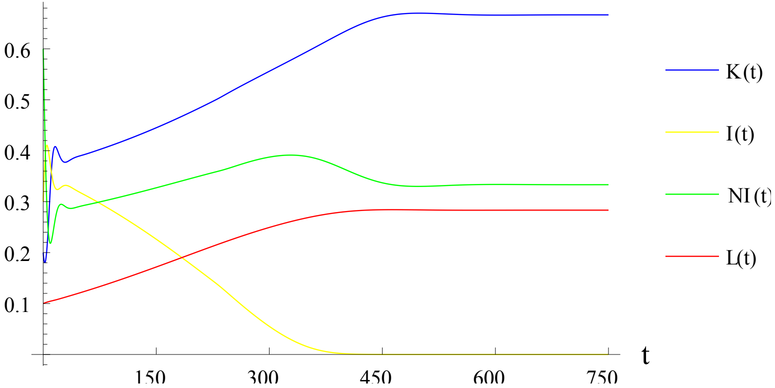


$b = 0.05$, $d = 0.05$, $m_2 = 0.01$



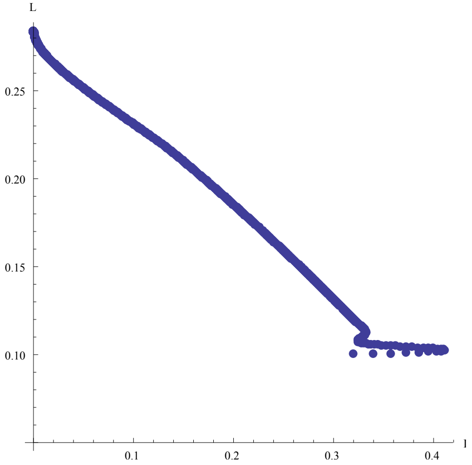
Results

$b = 0.5$, $d = 0.05$, $m_2 = 0.01$



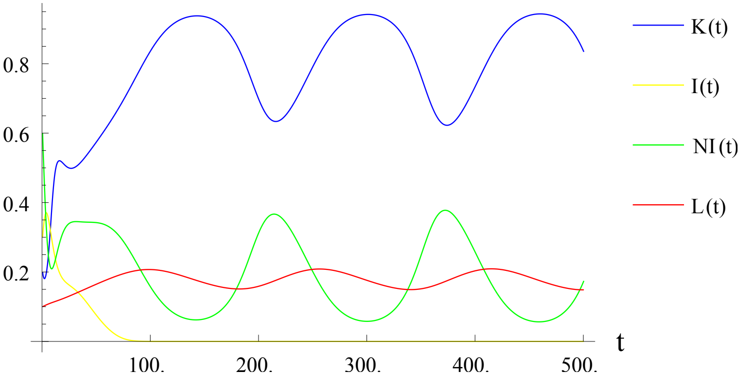
Results

$b = 0.5$, $d = 0.05$, $m_2 = 0.01$



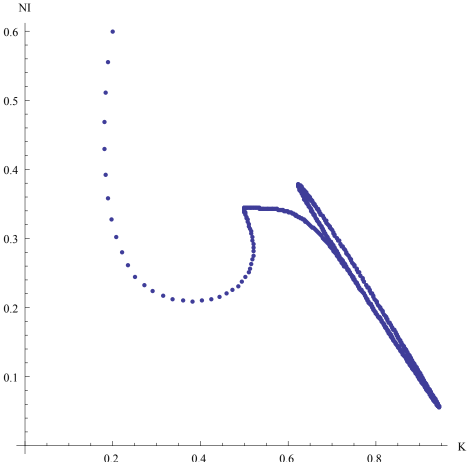
Results

$b = 0.8$, $d = 0.05$, $m_2 = 0.01$

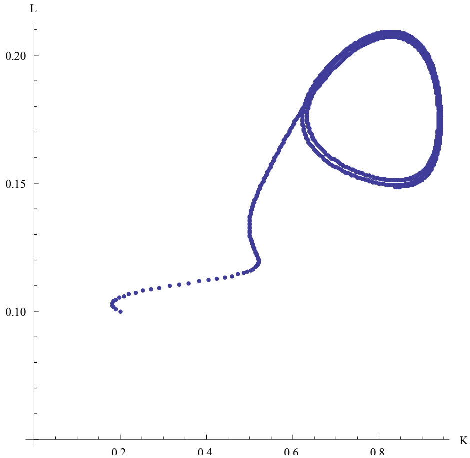


Results

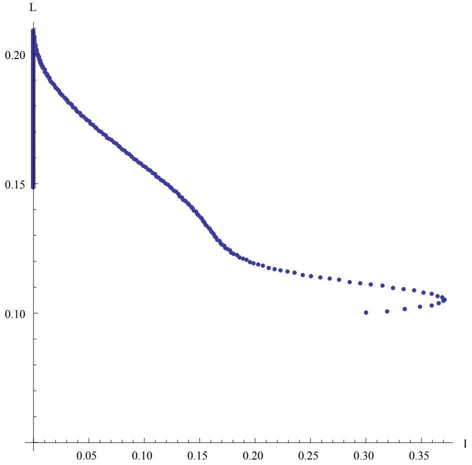
$b = 0.8,\ d = 0.05,\ m_2 = 0.01$



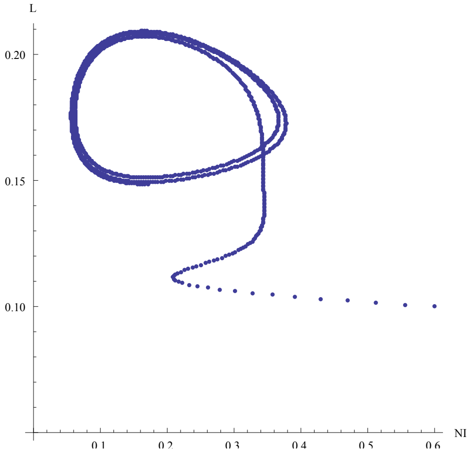
$b = 0.8$, $d = 0.05$, $m_2 = 0.01$



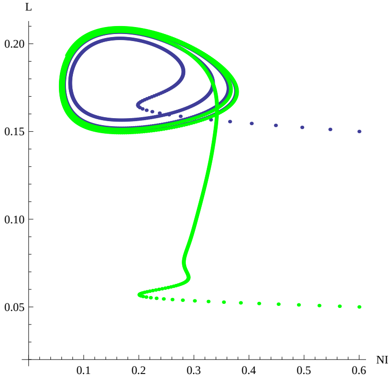
$b = 0.8$, $d = 0.05$, $m_2 = 0.01$



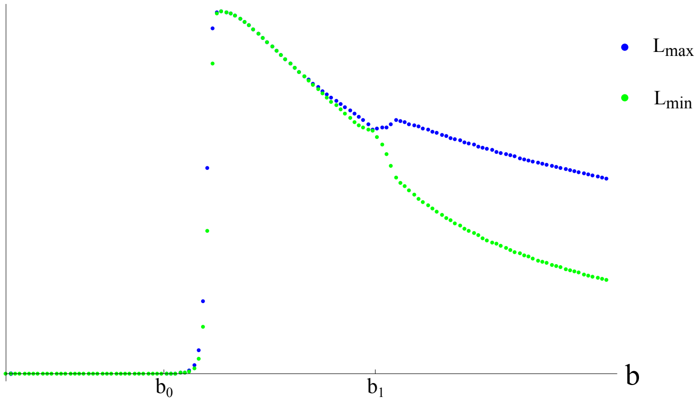
$b = 0.8$, $d = 0.05$, $m_2 = 0.01$



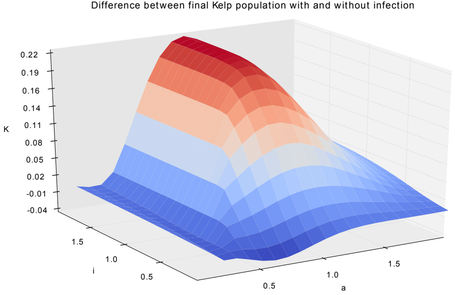
Different initial conditions

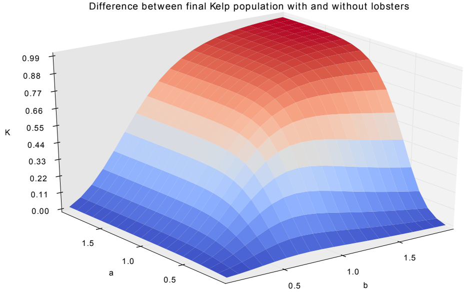


Bifurcation and onset of limit cycle



Magnitude of top-down control by pathogens versus predators





Conclusions

- The mathematical model describes well the dynamics of the system;
- The conditions required for the top-down control of the system by lobsters were determined;
- The epidemics exerts a top-down control on the urchin population, however the magnitude of this control is smaller than the control exerced by the apex predator.

Future projections of the work

- Evaluate fishing effort effect on the dynamics of the system.

References

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